LLNL-PRES-501654

FASTMath: An overview of numerical algorithms and software

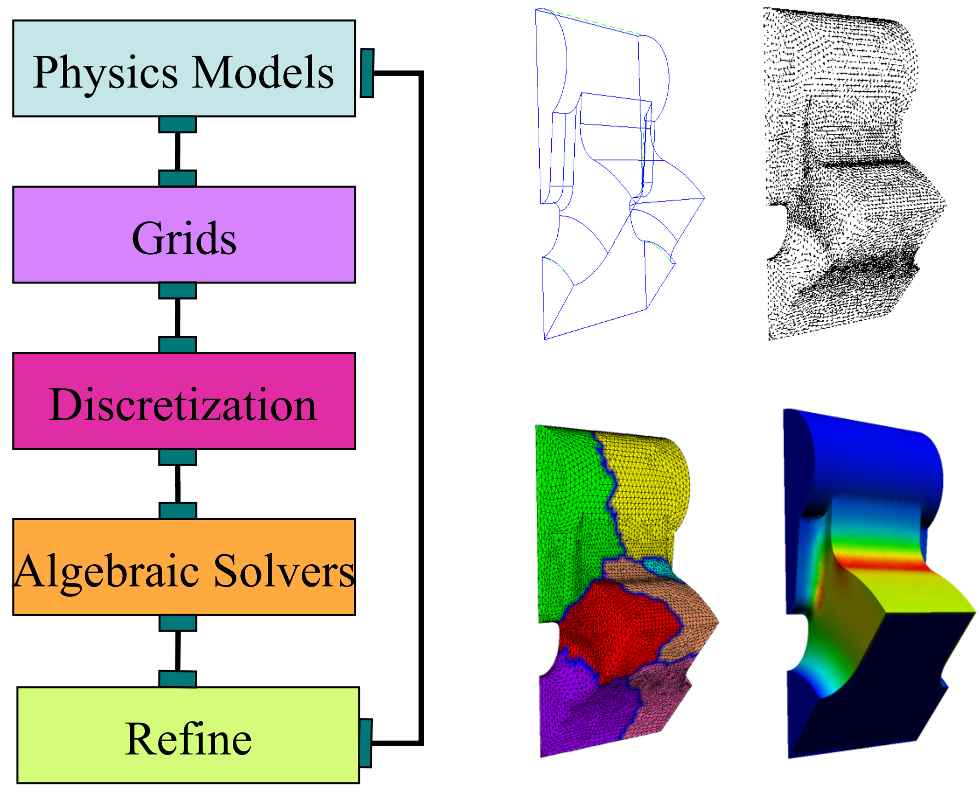
FASTMath Team

Lori Diachin, Institute Director

FASTMath SciDAC Institute

Mathematical algorithms and software are foundational to HPC simulations

- Develop a mathematical model of the phenomenon of interest
- Approximate the model using a discrete representation
- Solve the discrete representation
- Adapt and refine the mesh or model
- Couple different physics, scales, regions together



These steps require: CAD models, grid generation, high order discretizations, time integration techniques, linear and nonlinear solution of algebraic systems, eigensolvers, mesh refinement strategies, physics coupling methods, particle techniques, etc…

First consider a very simple example

- 1D rod with one end in a hot water bath, the other in a cold water bath
- Mathematical Model

$$\nabla^2 T = 0 \in \Omega$$

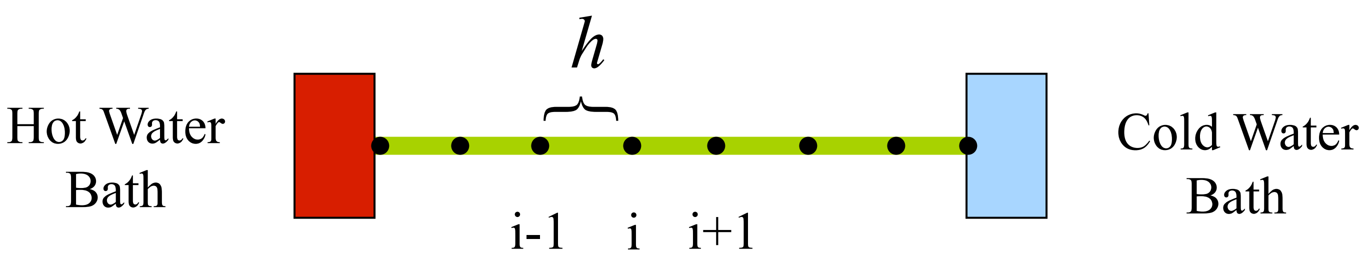
$$T(0) = 180^o \quad T(1) = 0^o$$

Discretizating the equations

- Approximate the derivatives in the continuous equations with a discrete representation that is easier to solve
- One approach: Finite Differences

$$\nabla^2 T \approx (T_{i+1} - 2T_i + T_{i-1})/h^2 = 0$$

$$T_0 = 180^o \qquad T_n = 0^o$$



Solve for the unknowns T_i

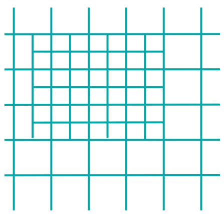
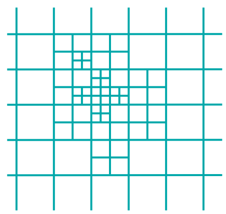
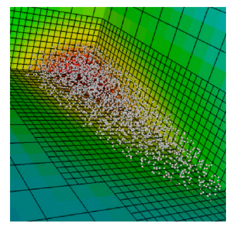
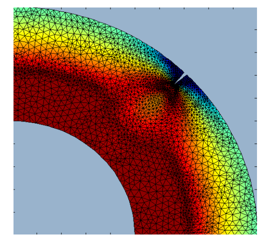
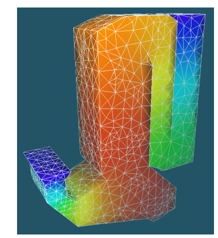
- Set up a matrix of the unknown coefficients
  - include the known boundary conditions
- Solve the linear system for T_i

$$\begin{pmatrix} 2 & -1 & 0 & \cdots & & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & 0 \cdots & 0 \\ & & \cdots & & & \\ 0 & \cdots & & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ . \\ T_{n-1} \end{pmatrix} = \begin{pmatrix} 180\, h^2 \\ 0 \\ 0 \\ . \\ 0 \end{pmatrix}$$

- Visualize and analyze the results

As problems get more complicated so do the steps in the process

- Different discretization strategies exist for differing needs
  - Efficiency
  - Flexibility

- Most problems are time dependent and nonlinear
  - Need higher algorithmic levels than linear solvers
- Increasingly combining multiple physical processes
  - Interactions require careful handling
- Goal-oriented problem solving required optimization, uncertainty quantification

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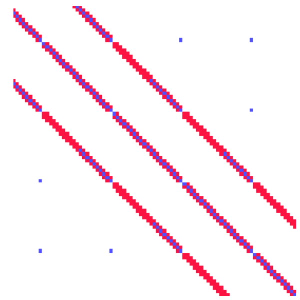
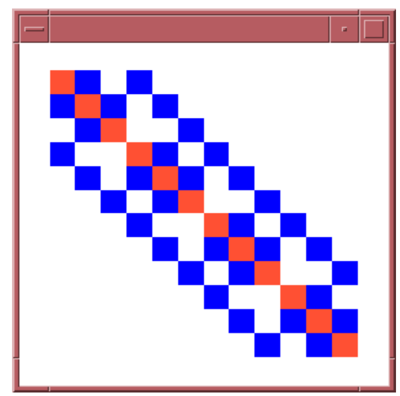
As problems grow in size so do the corresponding discrete systems

- Targeting applications with billions grid points and unknowns
- Most linear systems resulting from these techniques are LARGE and sparse
- Often most expensive solution step
- Solvers:
  - Direct Methods (e.g. Gaussian Elimination)
  - Iterative Methods (e.g. Krylov Methods)
    - Preconditioning is typically critical
    - Mesh quality affects convergence rate
- Many software tools developed at DOE labs deliver this functionality as numerical libraries
  - PETSc, Hypre, SuperLU, etc.

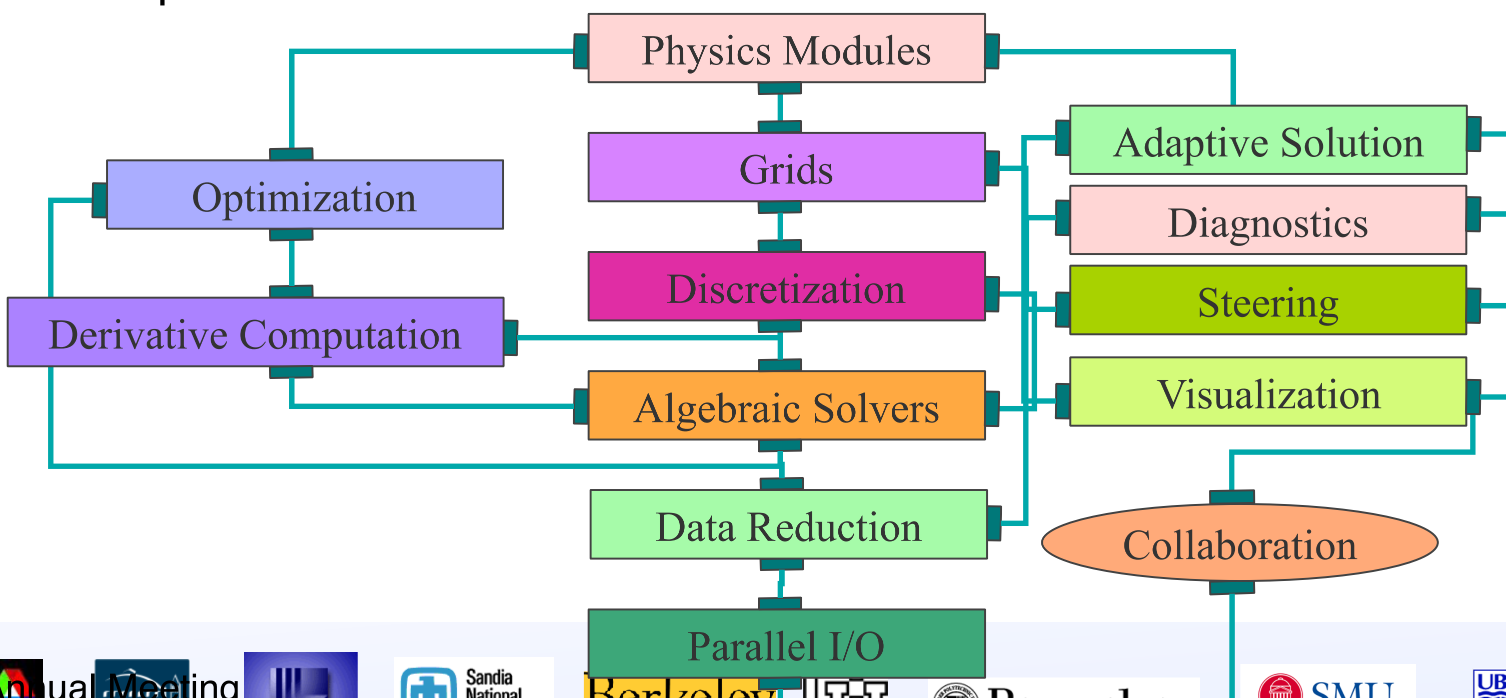




Modern scientific application development involves many different tools, libraries, and technologies

Observation: Exascale computing will enable high-fidelity calculations based on multiple coupled physical processes and multiple physical scales

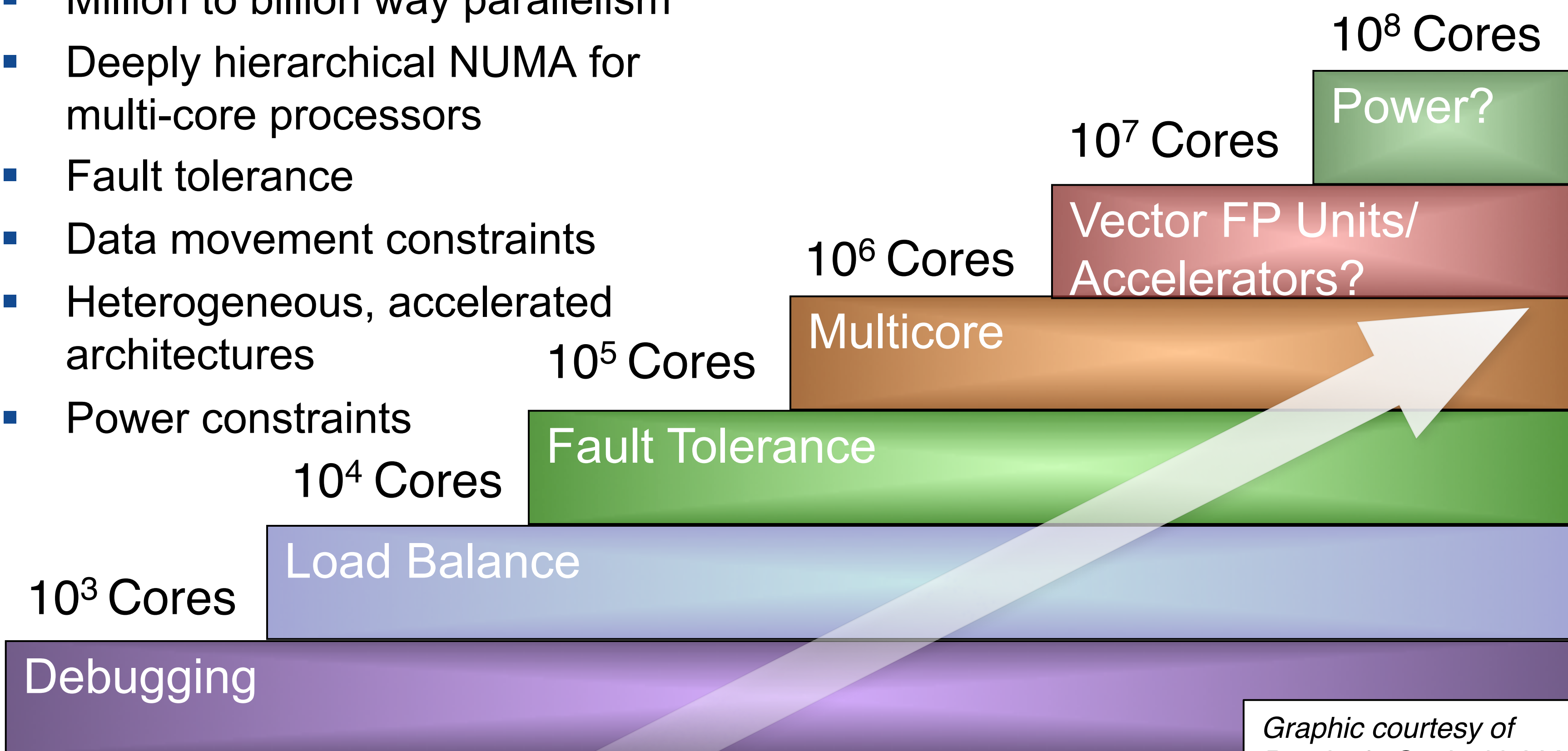
- Adaptive algorithms
- Composite or hybrid solution strategies
- High-order discretization strategies
- Sophisticated numerical tools



Modeling and simulation is significantly complicated by the change in computing architectures

Scientific computing software must address ever increasing challenges:

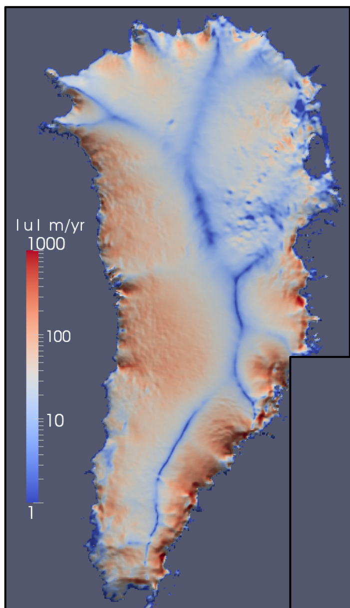
- Million to billion way parallelism
- Deeply hierarchical NUMA for multi-core processors
- Fault tolerance
- Data movement constraints
- Heterogeneous, accelerated architectures
- Power constraints

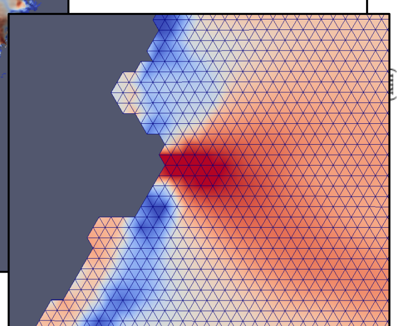


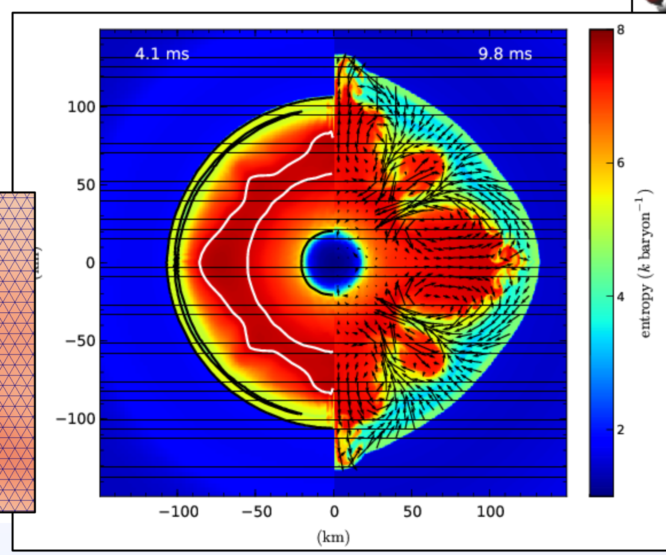
Graphic courtesy of Bronis de Supinski, LLNL

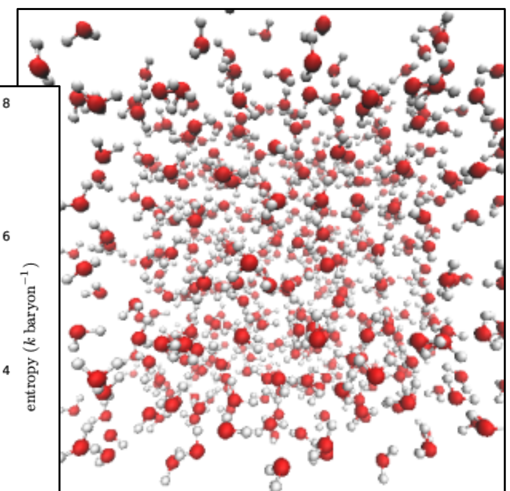
The FASTMath SciDAC project focuses on the development and use of mathematics software libraries

The FASTMath SciDAC Institute develops and deploys scalable mathematical algorithms and software tools for reliable simulation of complex physical phenomena and collaborates with DOE domain scientists to ensure the usefulness and applicability of FASTMath technologies

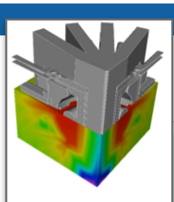






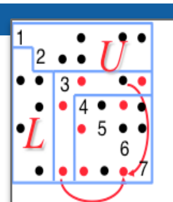


FASTMath encompasses three broad topical areas



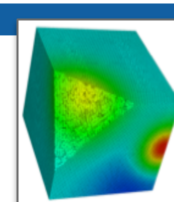
Tools for Problem Discretization

- Structured grid technologies
- Unstructured grid technologies
- Adaptive mesh refinement
- Complex geometry
- High-order discretizations
- Particle methods
- Time integration



Solution of Algebraic Systems

- Iterative solution of linear systems
- Direct solution of linear systems
- Nonlinear systems
- Eigensystems



High Level Integrated Capabilities

- Adaptivity through the software stack
- Management of field data
- Coupling difference physics domains
- Mesh/particle coupling methods

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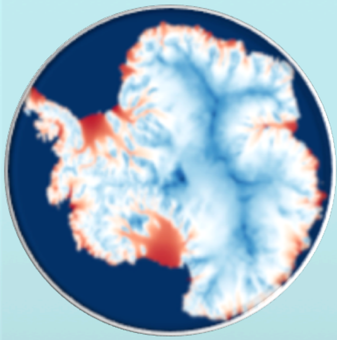
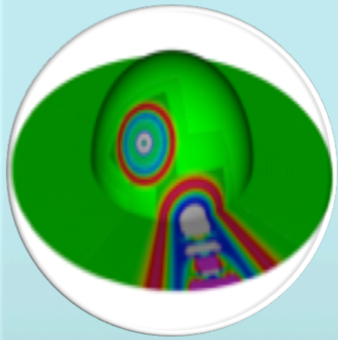
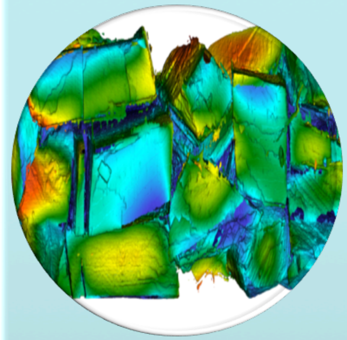
Structured grid capabilities focus on high order, mapped grids, embedded boundaries, AMR and particles



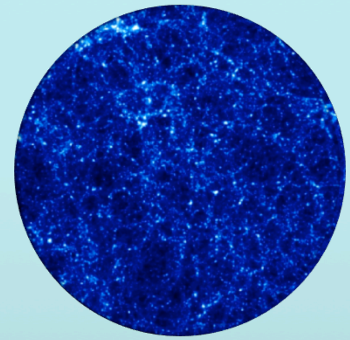
Structured AMR



Mapped-multiblock grids



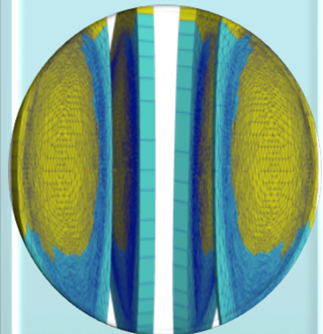
Embedded boundary methods



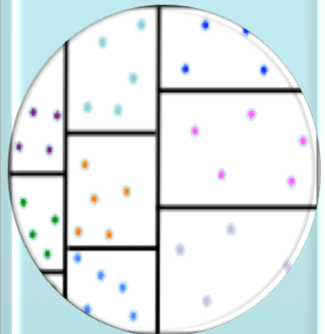
Particle-based methods

Application to cosmology, astrophysics, accelerator modeling, fusion, climate, subsurface reacting flows, low mach number combustion, etc.

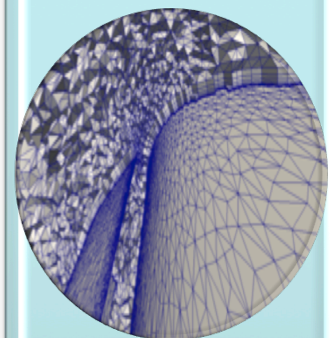
Our unstructured grid capabilities focus on adaptivity, high order, and the tools needed for extreme scaling



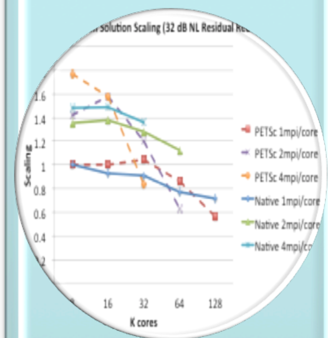
Parallel mesh infrastruct-ures



Dynamic load balancing



Mesh adaptation and quality control



Parallel performance on unstructured meshes

Architecture aware implementa-tions

Application to fusion, climate, accelerator modeling, NNSA applications, nuclear energy, manufacturing processes, etc.

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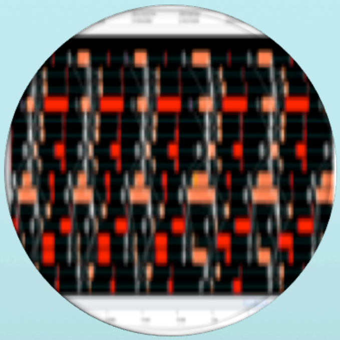
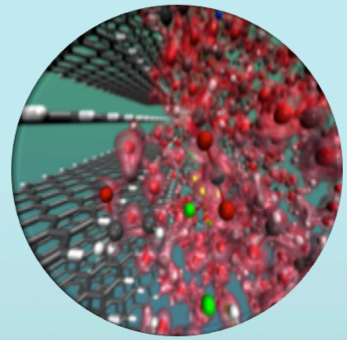
Our work on algebraic systems provides key solution technologies to applications

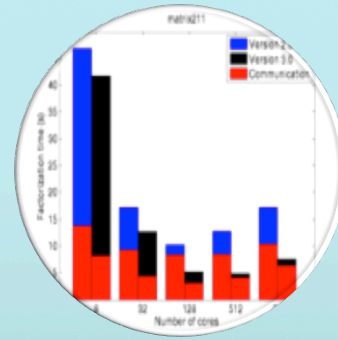


Linear system solution using direct and iterative solvers

Nonlinear system solution using acceleration techniques and globalized Newton methods



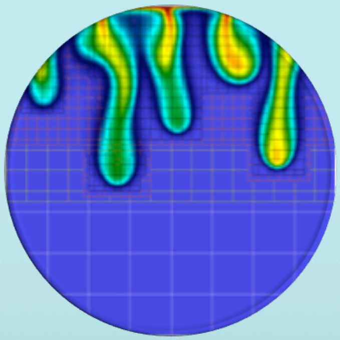
Eigensolvers using iterative techniques and optimization



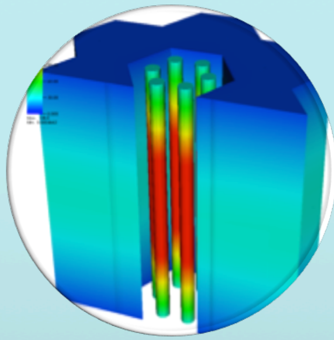
Architecture aware implementations

Application to fusion, nuclear structure calculation, quantum chemistry, accelerator modeling, climate, dislocation dynamics etc,

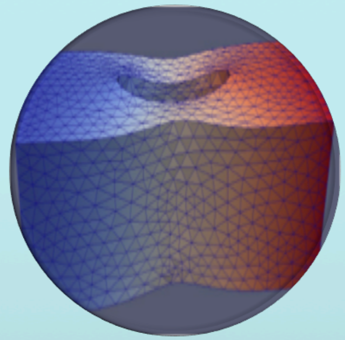
Integrating technologies is a key value added by the FASTMath Institute



Mesh/solver interactions



Mesh-to-mesh coupling methods



Unstructured mesh technologies into simulation workflows

Software unification strategies

Application to climate, plasma surface interactions, structural mechanics, nuclear energy, cosmology, fluid flow, etc.

FASTMath encompasses our algorithm development in widely used software

FASTMath Software

Structured Mesh Tools
- BoxLib (Ann Almgren)
- Chombo (Phil Colella)

Linear Solvers
- Hypre (Rob Falgout)
- PETSc (Barry Smith)
- SuperLU (Sherry Li)
- ML/Trilinos (Jonathan Hu)

Unstructured Mesh Tools
- PUMI (Seegyoung Seol)
- MeshAdapt (Mark Shephard)
- Sigma (Vijay Mahadevan)
- Mesquite (Lori Diachin)
- PHASTA (Ken Jansen)
- APF (Cameron Smith)

Nonlinear Solvers/Differential Variational Inequalities
- SUNDIALS (Carol Woodward
- PETSc (Barry Smith)
- NOX/Trilinos (Andy Salinger)

Partitioning Tools
- Zoltan (Karen Devine)
- ParMA (Mark Shephard)

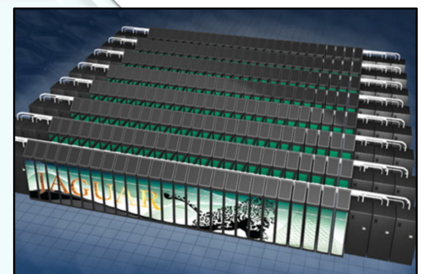
Eigensolvers
- PARPACK (Chao Yang)

Time Integrators
- SUNDIALS (Carol Woodward)
- PETSc (Barry Smith)

Our research to improve performance on HPC platforms focuses on both inter- and intra-node issues

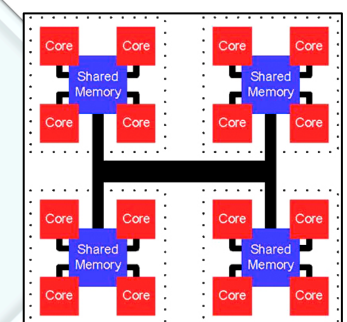
Inter-node: Massive Concurrency

- Reduce communication
- Increase concurrency
- Reduce synchronization
- Address memory footprint
- Enable large communication/computation overlap



Intra-node: Deep NUMA

- MPI + threads for many packages
- Compare task and data parallelism
- Thread communicator to allow passing of thread information among libraries
- Low-level kernels for vector operations that support hybrid programming models



We are developing new algorithms that address key bottlenecks on modern day computers

Reduce communication

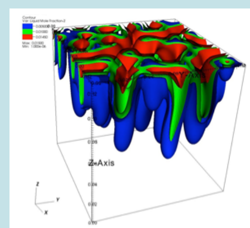
- AMG: develop non-Galerkin approaches, use redundancy or agglomeration on coarse grids, develop additive AMG variants (hypre) (2X improvement)
- Hierarchical partitioning optimizes communication at each level (Zoltan) (27% improvement in matrix-vector multiply)
- Relaxation and bottom solve in AMR multigrid (Chombo) (2.5X improvement in solver, 40% overall)
- HSS methods

Increase concurrency

- New spectrum slicing eigensolver in PARPACK (Computes 10s of thousands of eigenvalues in small amounts of time)
- New pole expansion and selected inversion schemes (PEXSI) (now scales to over 100K cores)
- Utilize BG/Q architecture for extreme scaling demonstrations (PHASTA) (3.1M processes on 768K cores unstructured mesh calculation)

Reduce synchronization points

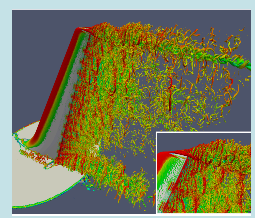
- Implemented pipelined versions of CG and conjugate residual methods; 4X improvement in speed (PETSc) (30% speed up on 32K cores)



Used in PFLOTRAN applications

Address memory footprint issues

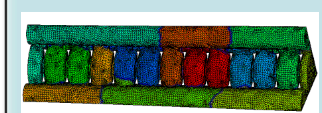
- Predictive load balancing schemes for AMR (Zoltan) (Allows AMR runs to complete by maintaining memory footprint)
- Hybrid programming models



Used in PHASTA extreme scale applications

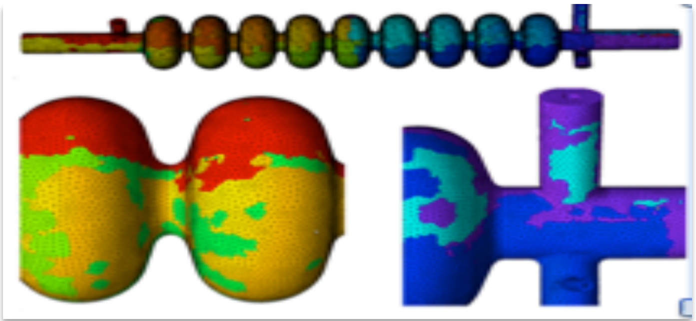
Increase communication and computation overlap

- Improved and stabilized look-ahead algorithms (SuperLU) (3X run time improvement)

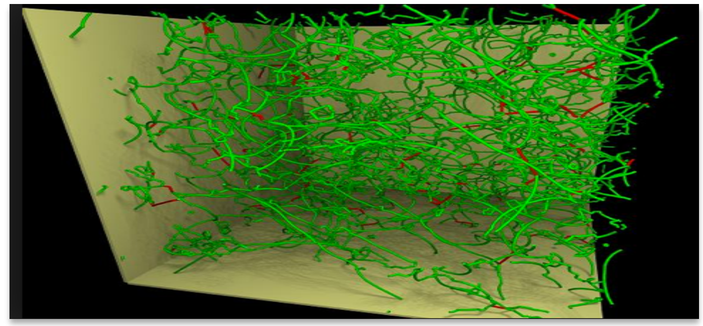


Used in Omega3P accelerator simulations

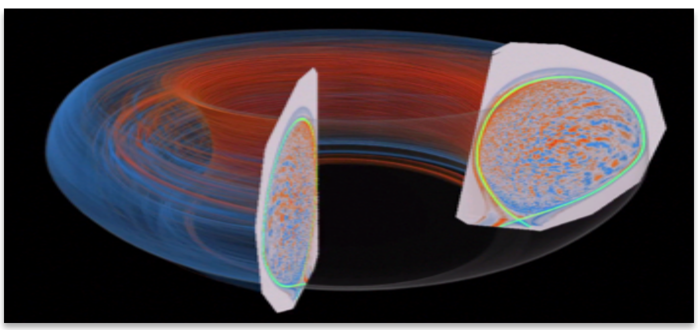
We have helped the application teams significantly reduce time to solution in their simulations



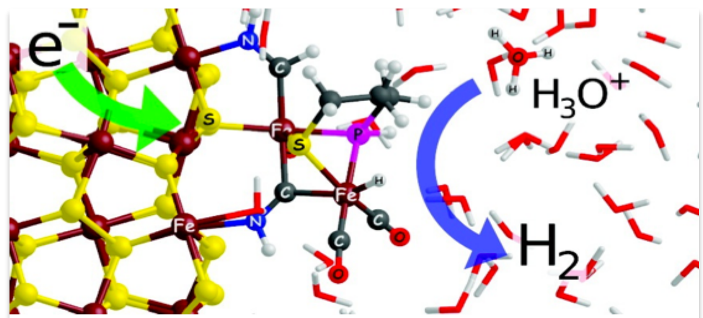
Sparse direct solves improve time to solution 20X for accelerators allowing 8 cavity simulation (Spentzouris)



Acceleration-based nonlinear solvers speed up dislocation dynamics 35-50%; multistage Runge-Kutta methods reduce time steps by 94% (Arsenlis)

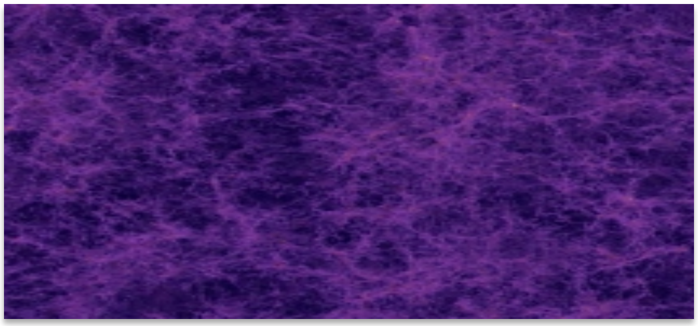


Sped up flux surface creation to improve 2D mesh generation in fusion application from 11.5 hours to 1 minute (Chang)

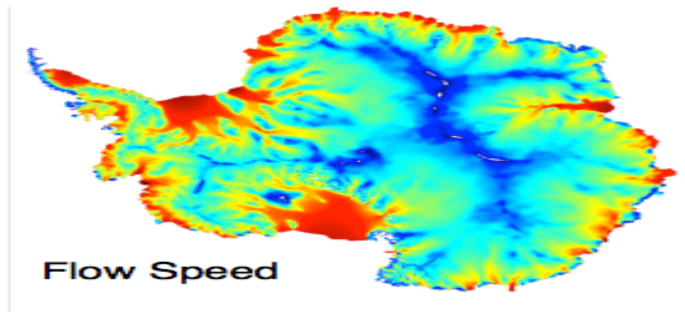


Sophisticated eigensolvers significantly improve materials calculations in many domains including ions in solution (Car), excited state phenomenon (Chelikowsky, Head-Gordon)

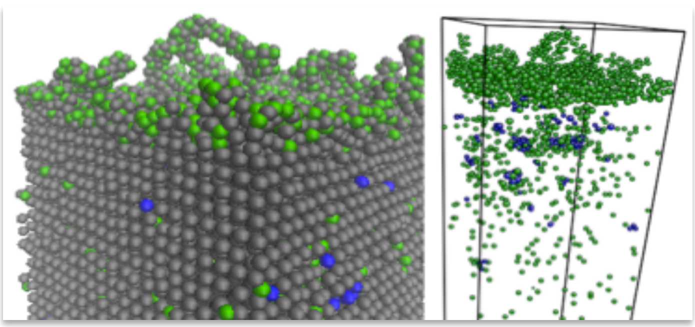
We have helped the application teams achieve unprecedented resolution and increased reliability



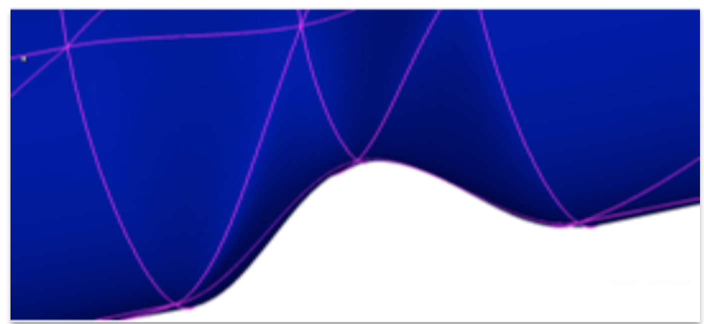
Astrophysics Lyman-α forest simulation at 4096^3 in an 80Mpc/h box; produced statistics at 1% accuracy for first time (Habib)



Predictions of grounding line match experiment for first time in ice sheet modeling due to AMR (Price)



Implicit ODE integrators combined with AMG linear solvers enables solution of 4D reaction-diffusion eqns for plasma surface interactions (Wirth)



High-order unstructured meshes for particle accelerators overcome mesh generation/ adaptation bottlenecks (Spentzouris)

The FASTMath team includes experts from four national laboratories and six universities

Lawrence Berkeley National Laboratory

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Phil Colella
Anshu Dubey
Dan Graves
Sherry Li
Lin Lin
Terry Ligocki
Mike Lijewski
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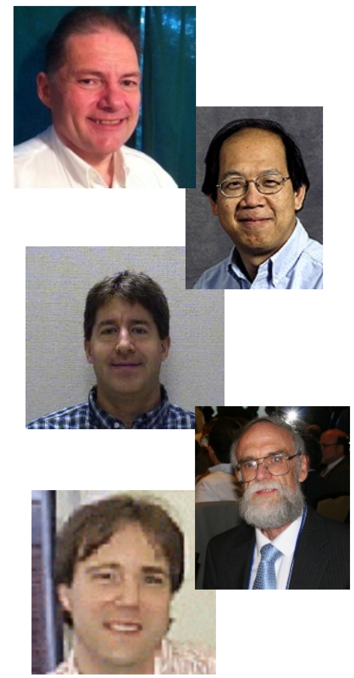

http://www.fastmath-scidac.org

Auspices and Disclaimer

Support for this work was provided through Scientific Discovery through Advanced Computing (SciDAC) program funded by U.S. Department of Energy, Office of Science, Advanced Scientific Computing Research

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.